Load Modulation for Damping of Electro-Mechanical Oscillations

Olof Samuelsson (Member)
IEA, Lund University, PO Box 118, SE-221 00 Lund, Sweden
Olof.Samuelsson@iea.lth.se

***Abstract*—Today it is feasible for utilities to control selected customer loads in a non-disturbing fashion. This is comparable to load shedding, but has a wider area of application. This paper focuses on direct load control or modulation for power system damping and is based on a field test of on-off control and modal analysis of DAE models of three test systems. Real power load controlled by bus frequency is found to be robust to changes in network topology. It is shown that on-off modulation is more effective than sinusoidal modulation for damping of large oscillations, but not for small oscillations. It proves difficult to give a general measure of the required amount of controlled load, since modulation of real power loads has an additive rather than multiplicative effect on power flows. Different structures of load control systems are listed. If the system is centralized, it is suggested that the distribution company owns and operates the system and that damping is sold as ancillary service.**

Keywords: small-signal stability, damping, electro-mechanical oscillations, modal analysis, load modulation, load control, on-off control

I. INTRODUCTION

Today the need for larger power transfers in many parts of the World increases faster than the power systems can expand. This leads to reduced margins to various stability problems. Traditional network reinforcement is now less practical for a number of reasons. Instead it becomes highly interesting to better utilize the existing thermal transfer capacity through control options. For this purpose, the EPRI-supported *WAMS* (Wide Area Measurement System) and *FACTS* projects were launched to develop information technology for measurement, communication and control using high voltage power electronics. At the same time systems for utility control of selected customer loads were installed as one of several *DSM* (Demand Side Management) activities. Although developed mainly for steady state purposes such as *peak shaving*, load control is an asset also when resolving *dynamic* problems.

This paper focuses on the use of load control or *load modulation* for damping of electro-mechanical oscillations. Although mentioned already in 1968 [1], this has not been studied until recently [2-9]. In a recent overview of damping methods [10], indirect load modulation through voltage variations is mentioned, but not this more direct type of load control. It can be viewed as a sophisticated alternative to *load shedding*, which is intentional blackout of entire districts by opening of one or more MV circuit breakers. This can only be used as a last resort to save system operation in emergency situations, but under-voltage and underfrequency load shedding are well-known measures against load-frequency control problems [11] and voltage collapse [12] respectively. Transient angle instability can also be avoided through load shedding [11]. In all these cases, modern load control could be used more frequently, to a more variable extent and with much less negative impact on customers than load shedding. When using load modulation for damping purposes, key questions are which loads to control, with what strategy and what performance can be achieved? This paper aims at answering these and other closely related questions.

II. TEST SYSTEMS AND STUDIES

The conclusions reported here are based mainly on studies of four test systems that are all described in [4]. Additionally, power system dynamic measurements from the 230 V mains have been analyzed [13].

A. *Real single mode system*

A field test of on-off load control was arranged in the system shown in Fig. 1. It features a 0.88 MVA hydro generator weakly connected to a strong bus. The heating elements of four 5kW heating fans, were controlled by thyristor switches that replaced the thermostats. Machine angle relative to the strong bus was synthesized from current and voltage measurements in one phase. The control law switched the load on, when the time derivative of the synthesized angle exceeded a threshold $\Delta\omega_{on}$ and switched off when it fell below zero. This system was also studied using time simulations of a detailed and simplified model. For more details, see [4, 5].

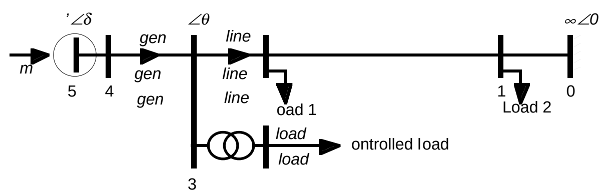

Fig. 1. Real single machine infinite bus system used for field test.

B. *Generic multimachine system*

Equation (1) shows the linearized multimachine version of the swing equation, and has real power load as input and bus voltage phase angle as algebraic variable and output. This model was used for analytical investigations, but the close agreement with a spring-mass model also permits use of mechanical analogies and interpretations.

$$\begin{bmatrix} M & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} \Delta\omega \\ \Delta\delta \\ \Delta\theta \end{bmatrix} = \begin{bmatrix} 0 & K_{\delta\delta} & K_{\delta\theta} \\ I & 0 & 0 \\ 0 & K_{\theta\delta} & K_{\theta\theta} \end{bmatrix} \begin{bmatrix} \Delta\omega \\ \Delta\delta \\ \Delta\theta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -I \end{bmatrix} \Delta P_{load} \quad (1)$$

C. *WSCC9 and NORDIC32*

The two standard test systems shown in Fig. 2 were employed for numerical analysis. The WSCC9 [14] was chosen for being simple yet non-trivial, while the CIGRE

NORDIC32 [15] is more complex and realistic. Differential-algebraic equation (DAE) matrix models of both systems were created from full non-linear models in EUROSTAG [16] using the standard linearization feature and export to Matlab. Thanks to this, comparison between linear analysis and non-linear simulation is straightforward and full agreement is expected. Lines, transformers, generators and associated controls were modeled as simple as possible for the WSCC9 [3], and more realistically for NORDIC32 [6]. The numbers of states in the two models are 42 and 545 of which 9 and 324 are dynamic. These systems were analyzed using modal analysis of the DAE model. Based on this, the locations for two proportional damping controllers were chosen. Local bus frequency and shaft speed deviation at the closest machine were compared as input signal.

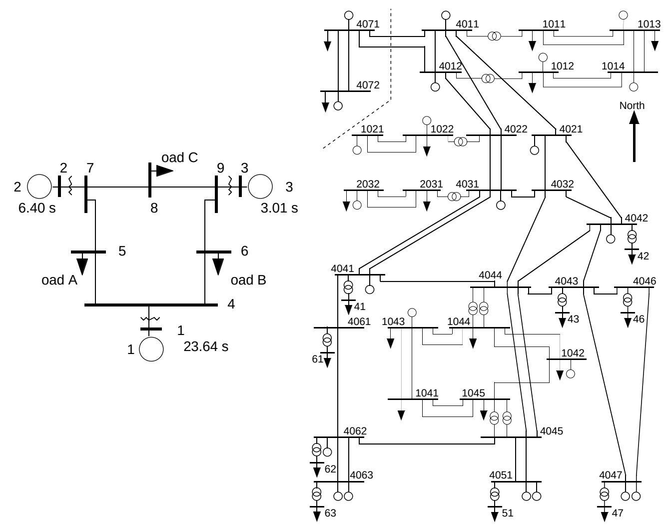

Fig. 2. Single line diagram of WSCC9 with inertia constants on 100 MVA base (left) and NORDIC32 (right).

D. 230V measurements

Local bus frequency measurements were obtained from a power quality monitor at the 230V level. Frequency data was collected with a rate of 50 samples per second at different times of day at two occasions. This was done to get an idea about the oscillation activity at one point in the NORDEL system, but also about disturbances and SNR. A general observation reported in [13] was that it is difficult to detect this activity during normal operation.

III. TYPE OF LOAD TO MODULATE

During electro-mechanical oscillations, kinetic energy swings between the rotors of the participating generators. This leads to variations mainly in the flows of real power. Any actuator that can affect these flows is suited for damping. This includes HVDC, SMES, reactive series and shunt compensation and generators that are all described in [10]. Control of real power injections is particularly powerful and therefore SMES and HVDC links are efficient actuators for damping. Varying network impedances using reactive series compensators also has a strong effect, but this depends on magnitude and direction of the power flow. Reactive shunt compensators such as the SVC are also used for damping. Like a PSS, an SVC affects the real power flow both directly and through modulation of voltage sensitive loads. These two effects add, but depend on flow direction and voltage sensitivity of loads and may even change sign [10]. Real power modulation is more robust to changing line flows than control of reactive series and shunt compensation [2], and therefore real power loads are chosen for damping modulation. All results referenced here model controlled load as a variable injection of real power at transmission level. Low voltage loads are also expected to be effective, but the distribution system may affect the performance.

Annoyance from control actions is important and is reduced if the load has energy storage. Well-suited real power loads are thus most heating loads such as water heaters and boilers. Resistive space heating can also be used even if the storage is external. The storage may also be magnetic as in a SMES [8, 18] or a battery as in a UPS or a future charging station for electric vehicles. All these components seem well suited for load control. The latter are particularly interesting as they have a power electronic grid interface that easily can be made available for the new control task. For thermal loads the thermostat may have to be complemented by a simple thyristor switch as in [5]. Due to their high penetration, cooling loads such as refrigerators and air conditioners are well suited for peak shaving [7] and also have potential for use in damping schemes. Their motors may, however, be damaged from frequent load control actions.

IV. WHERE TO MODULATE LOAD

The best location for load control is determined through modal analysis of a linearized dynamic model of the power system. In [3] it is shown that DAE representation of the model is particularly useful, since its eigenvectors hold information related not only to generators but also to the buses. If the eigenvector can be scaled so that the phase angles of the bus-related elements are grouped around zero and 180°, these can be made approximately real. This real magnitude data can then be superimposed as positive and negative vertical bars on the network topology as in Fig. 3. It shows controllability of real power for the slowest electro-mechanical mode in the WSCC9.

This visualization gives a general idea of which areas are more or less suited for load modulation when targeting individual modes. To some extent it allows interpolation between existing buses as illustrated by the dashed lines. In [17] it is concluded that, to be effective, real power modulation in a two area system should be done at a small *mass-scaled electrical distance* from the swinging machines of the two areas. Furthermore, at one point between these areas, the modulation is ineffective since the controllability is zero here. This agrees well Fig. 3, but the visualization also generalizes this to multi-machine systems with meshed networks. If several modes are targeted, different geographical variation of mode controllability must be taken into account when deciding where to control load.

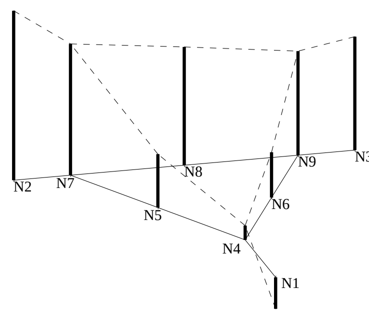

Fig. 3. 3D-view of the network topology of the WSCC9 system with geographical real power controllability of the slowest mode superimposed.

V. ON-OFF OR CONTINUOUS CONTROL

For the individual load, simply switching it on or off is the simplest. This can be actuated by a relay, possibly of semiconductor type and only requires a binary signal and thus very little communication bandwidth. This is easily extended to a large number of loads that are switched simultaneously. If instead the loads are controlled individually in a coordinated way, it is possible to create a control action of variable magnitude. This applies also for a single load with a power electronic grid interface. It can be continuously controlled to draw power ranging from a maximum down to zero or even negative value. For damping of large oscillations on-off control is superior. The resulting square-wave signal is equally effective as the sinusoidal signal corresponding to its *fundamental* component, as described and exemplified in the discussion of [18]. As shown in Fig. 4, the peak value of the fundamental is $4/\pi$=127% of the square-wave amplitude. For a certain actuator rating, on-off control is thus 27% more efficient than sinusoidal modulation.

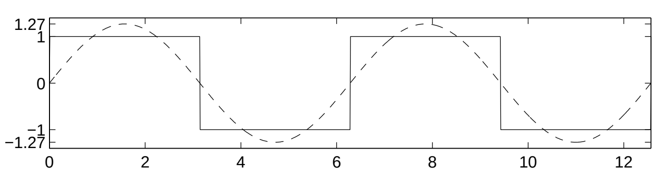

Fig. 4. The fundamental of a square-wave has 27% higher amplitude.

On-off control modulates a fixed quantity of load, which gives a *linear* decay of the oscillation as the same amount of swing energy is removed each cycle. A large control signal thus gives a fast decay. A small control signal, on the other hand, can be compensated by a longer duration of the modulation sequence. A linear controller with output limits is the most realistic alternative to on-off control. As long as the output limit is active, its behavior is similar to on-off control yet with a smoother transition between the two extremes. When the controller enters its linear range, the oscillation decays exponentially to zero. With on-off control, the oscillation can only be damped down to a certain level–a *deadband*–where the control actions should stop. If this is not done, the system enters a *limit cycle*, where the damping controller itself sustains an oscillation as described in [5] where a systematic method to select the deadband is given. For reduction below the deadband, a smaller control signal magnitude is needed. This is produced automatically by the linear controller. The key parameter for small signal behavior of the linear controller is the gain.

Most loads are expected to be on-off controlled. Temporarily turning a load off is usually acceptable. But the fact that only a part of the targeted loads can be expected to be on when control actions are required, leads to uncertainty in the resulting control signal magnitude. But modulation for damping has the advantage that switching loads on and off is equally effective. Therefore, if it only is acceptable also to switch loads on, *all* loads can be used. Furthermore, load modulation for damping is not one-shot but a sequence of repeated switchings, cycling between on and off. The first switching then gives a synchronization so that all loads are then either on or off as in Fig. 5. Subsequent switchings apply to all loads with a resulting maximum impact and no uncertainty.

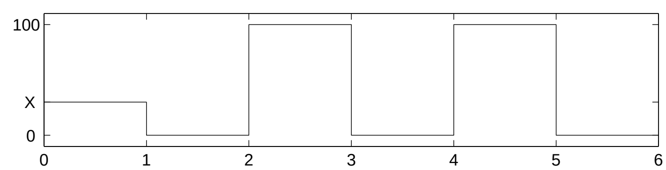

Fig. 5. Switching sequence with x% of controlled load initially on.

VI. AMOUNT OF LOAD TO MODULATE

The amount of controllable load required for oscillation damping is determined by the expected magnitude of oscillations and by the time available for reducing this down to an acceptable level. Additionally, the inherent damping in the system and the impact that the loads have on the targeted swing mode – their mode controllability – affect the required amount. The question is thus case-specific and some numbers from realistic systems are mentioned here:

• The field tests reported in [5], used 20 kW of controlled load in a system with 0.8 MW generation operated at 50 % output and some 100 kW other loads. This gave a reduction in the time needed to damp oscillations from 0.1 MW amplitude to 0.06 MW from 7.9 s to 1.9 s.

• The FennoSkan HVDC link between Finland and Sweden has a rating of 500 MW. Its damping controller uses frequency difference as feedback signal and has an output limited to 50 MW. This is known to be very powerful in the NORDEL system.

• The study on load modulation in the WSCC system [7], found that ±450 MW real power modulation was comparable to ±500 Mvar reactive power modulation in that they both made a 400 MW transfer capacity increase possible. It also showed that if load was controlled at ten different sites, the total amount of controlled load could be reduced by 30%.

• Control of approximately 5% of the total load (37000 MW year 2000) in the Hydro Quebec system is used in [9] to survive a critical contingency.

Note especially, that the effect of real power modulation is *additive* to other flows, rather than multiplicative. It is therefore not possible to determine a typical percentage of controlled MW relative to other MW such as generator rating or operating point. Such a number is meaningful only for the individual system.

VII. FEEDBACK SIGNAL AND GAIN

In all feedback control systems, the choice of measurement signal is important. Damping controllers use signals that are related to the oscillating generator rotors or to the resulting power variations in the lines between them. This makes shaft speed, machine angle and line flow relevant signals, but also phase angle and bus frequency are candidate signals. As previously mentioned, real power modulation should be done close to areas with swinging generators. This is also where the mode observability is greatest for phase angle and bus frequency. Fig. 6 illustrates the geographical variation of the mode observability of phase angle for one mode in the WSCC9 system. Comparing this with the real power mode controllability in Fig. 4 reveals a striking agreement. This is no coincidence; in fact it is rather a rule.

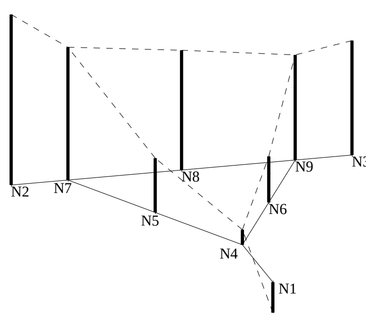

Fig. 6. Mode observability of phase angle for the slowest electro-mechanical mode of the WSCC9 system.

For the generic multimachine system in Section II, it can be shown (see Appendix A) that for each mode the vector of mode observability of phase angle is proportional to the vector of mode controllability of real power. The proportionality constant is a negative real number. The same applies to bus frequency as it is simply the time derivative of the phase angle. Multiplication of the controllability and observability vectors gives the eigenvalue sensitivity to the gain in a controller using local bus frequency as input and real power injection as output. This sensitivity is real and negative for all modes and all controller locations, which means that introducing such a controller has positive or no effect on damping of all modes. Two root locus plots from the application of such a damping controller in the WSCC9 and NORDIC32 systems are shown in Fig. 7. They confirm that the eigenvalues of the electro-mechanical modes that can be affected all initially move straight to the left.

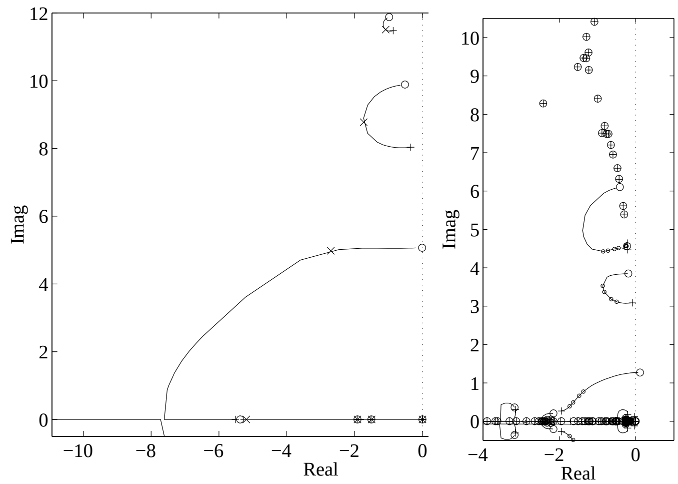

Fig. 7. Root locus plots for a damping controller with real power output and bus frequency input at bus N8 in WSCC9 (left) and at bus 63 in NORDIC32 (right).

Increasing the gain gives moves the targeted eigenvalues further to the left. Fig. 8 shows what performance that can be obtained with two damping controllers at buses 47 and 63 using bus frequency as input and at buses 47 and 51 using machine frequency respectively. Larger gains, however, change the eigen-vector shape and cause the root locus to turn. Mode observability and mode controllability are also influenced, but there is a chance that they are equally affected. This may explain the better robustness to changes in network topology when real power modulation is controlled by bus frequency as compared to machine frequency of the closest generator. Fig. 8 illustrates this by showing eigenvalue locations in the two cases for NORDIC32 with the double line N4044-N4045 in and out of service. The gain selection is based on the line being out of service, and it can be seen that putting the line back in service reduces damping in the machine frequency case, while it is has little effect in the bus frequency case.

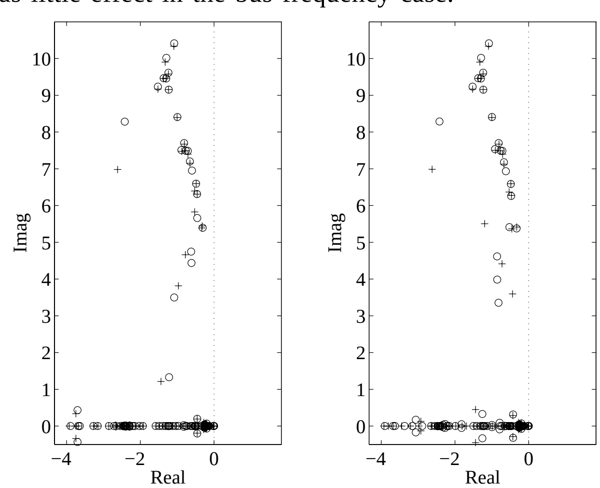

Fig. 8. Eigenvalues with line in (+) and out (o) of service with bus frequency (left) and machine frequency (right) as measurement signals for two controllers at buses 47 and 51 and 47 and 63 respectively.

VIII. FEEDBACK LOOP STRUCTURES

It is thus clear that a damping system based on real power injection proportional to local bus frequency variation is effective. It has also been shown where control actions and measurements should be performed, but only where in the meshed transmission system. The distribution system adds another dimension and it has to be decided if HV or LV loads should be used for control. Additionally, the measurement signal can be taken either on the HV level at the top of the distribution system, or near the LV load. Three combinations with different properties are outlined below.

Control of HV load using HV measurements

If electric boilers for district heating can be modulated in the appropriate time scale, they would be very attractive for load control for several reasons. As their rating is high and they are connected at HV or MV level, their impact on electro-mechanical oscillations is expected to be high. The control can be based on local measurements near the transmission system. Bus frequency at this level has both high modal content and high SNR. The only problem is that there are few loads of this type.

Control of LV load using HV measurements

LV loads that can be used for load control are abundant and can be found at every customer. This gives a high total rating of the distributed actuator. By using the communication installed for peak shaving, these loads can be remote controlled from the substation near the transmission system. This is also where measurements are done, with the advantages described above. Complexity is, however, higher for this structure than if few high voltage loads are used.

Control of LV load using LV measurements

The bus frequency signal is theoretically available, not only in the transmission system, but also throughout the distribution system. If the electro-mechanical oscillations could be detected in frequency measurements at the LV level, communication is not needed and the controller could be integrated into the individual appliance. If coordination of the autonomous controllers can be avoided, this implementation of load control for damping is very attractive.

All the results based on multimachine system models referenced here fall in the first category. The field test uses an LV load, but at the same time the generator is also located in the distribution system.

The measurements at 230V reported in [13] were aimed at detecting modal activity at an LV bus. They exhibited a low SNR, but indicated activity at frequencies characteristic of known swing modes in the NORDEL system. But since this system is well damped, the low SNR is only natural and when oscillations occur it is expected to be considerably higher. The most important conclusion was that detecting oscillations from frequency measurements requires high resolution.

The organizational aspects of the three structures are widely different. The electric district heating boilers in Sweden are typically owned and operated by distribution companies. If such a boiler or a similar installation could be used for damping, it is merely a matter of recovering the equipment costs. This is fairly straightforward if damping were an ancillary service that the distribution company could sell.

In the second case with centralized control of LV loads, more parties are involved. Contracts for interruptible power are available and should not pose any problems. When all customers now can choose which company to buy electric power from, some distribution companies object to load control that when they interrupt power delivery, they do not know from which retailer. Retailers also want to avoid being associated with interruptions. Both these problems are solved if the wire company itself is responsible for the load control and handles the compensation of on one side the customers and on the other side the transmission system operator. This structure allows the load control system to be used for other purposes such as peak shaving, voltage stability or load-frequency control. It is then natural that all these activities are coordinated by the distribution company.

In the third case, the damping equipment is totally distributed and autonomous. If the damping controller is built-into the appliances, these can be subsidized like low energy lighting. However, the need for coordination with this concept is unclear and multi-purpose load control is more difficult to achieve since all control actions must be based on locally available signals.

XI. CONCLUSIONS

Control of customer loads is an option that is not utilized to its full potential today. First it should be used to improve existing applications of load shedding. Next it can be used for more demanding purposes such as load modulation for damping. An important difference is the need for feedback control rather than manual control. Important issues are controllability, observability and control strategies. Real power loads are chosen for control as real power modulation is additive to the power swings that occur during oscillations. At the same time, the additive property makes it difficult to quantify in general terms how much load needs to be controlled. This is case-specific and is determined by mode controllability, existing and desired damping. Controlling real power in proportion to local bus frequency is shown to be robust to network topology changes. This is most likely due to the agreement between geographical variation in real power mode controllability and bus frequency mode observability, which are proven identical. On-off modulation is found to more effective and simpler to organize than linear control, but cannot be used to remove oscillations below a certain magnitude. The currently most important step towards implementing damping systems based on load control, is to introduce clear incentives. These need to be differently designed depending on if the system is centralized and owned by the distribution company or if the damping controller is integrated with individual appliances like water heaters, refrigerators or air conditioners.

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Olof Samuelsson (M'98) was born in Västerås, Sweden in 1966. He received his M.Sc.E.E. degree from Lund Institute of Technology in 1989. During 1990 he worked as a research engineer at the University Hospital of Lund. Olof Samuelsson received his Licentiate and Ph.D. degrees in 1994 and 1997 respectively from Lund Institute of Technology, Department of Industrial Electrical Engineering and Automation, where he now holds a position as postdoctoral fellow. During 1999 he spent six months as visiting researcher at Powertech Labs Inc. in Surrey, BC, Canada, and six months at the Technical University of Delft in Delft, Netherlands.

APPENDIX A.

Starting from (1), eliminate algebraic states and introduce new states $\tilde{\omega} = \sqrt{M}\omega$ and $\tilde{\delta} = \sqrt{M}\delta$:

$$\tilde{A} = \begin{bmatrix} 0 & M^{-1/2}KM^{1/2} \\ I & 0 \end{bmatrix}; \quad \tilde{B} = \begin{bmatrix} M^{-1/2}K_{\delta\theta}K_{\theta\theta}^{-1} \\ 0 \end{bmatrix};$$

$$\tilde{C} = \begin{bmatrix} 0 & -K_{\theta\theta}^{-1}K_{\theta\delta}M^{-1/2} \end{bmatrix}; \ \tilde{D} = \begin{bmatrix} K_{\theta\theta}^{-1} \end{bmatrix} \text{ where } K = K_{\delta\delta} - K_{\delta\theta}K_{\theta\theta}^{-1}K_{\theta\delta}$$

If the right and left eigenvectors Φ_i and Ψ_i of $\tilde{M}$ belonging to eigenvalue λ_i are partitioned into ω and δ parts, the following equations apply:

$$\text{Right eigenvectors: } \Phi_{i\omega} = \lambda_i \Phi_{i\delta}; \ 0 = \left(M^{-1/2}KM^{1/2} + \lambda_i^2 I\right)\Phi_{i\delta};$$

$$\text{Left eigenvectors: } \Psi_{i\delta} = \lambda_i \Psi_{i\omega}; \ 0 = \Psi_{i\omega}\left(M^{-1/2}KM^{1/2} + \lambda_i^2 I\right);$$

As the parenthesized matrices are symmetric, $\Phi_{i\delta}^T$ and $\Psi_{i\omega}$ are equal except for a (complex) scaling factor ξ_i. If $\arg\xi_i = -\arg\lambda_i$, $\Psi_i\Phi_i$ is real, which is standard. Both Ψ_i and Φ_i can now be expressed in $\Phi_{i\delta}$, λ_i and $\rho = \text{abs}\,\xi_i$. Phase angle mode observability for mode *i* is now:

$$\tilde{C}\Phi_i = -K_{\theta\theta}^{-1}K_{\theta\delta}M^{-1/2}\Phi_{i\delta}$$

Similarly the real power mode controllability is:

$$\left(\Psi_i \tilde{B}\right)^T = \rho_i \lambda_i^{-1} K_{\theta\theta}^{-1} K_{\theta\delta} M^{-1/2} \Phi_{i\delta} = -\rho_i \lambda_i^{-1} \tilde{C}\Phi_i$$

As these quantities differ only by a scalar, the geographical variation is the same.