Appendix D

Sideband Production in RMS Calculations

This appendix relates to Chapter 6, §6.2.

The trigonometric identities

$$\sin(x)\sin(y) = \tfrac{1}{2}[\cos(x-y) - \cos(x+y)] \tag{1A}$$

$$\sin(x)\cos(y) = \frac{1}{2}[\sin(x-y) + \sin(x+y)] \tag{1B}$$

occur repeatedly in the calculation of rms (root mean square) quantities. Consider, for example, an unmodulated synchronous current

$$i_0(t) = I_0 \sin(\omega_n t) \tag{2}$$

and its square

$$i_0^2(t) = I_0^2 \sin^2(\omega_n t) \tag{3A}$$

$$= \tfrac{1}{2} I_0^2[\cos(\omega_n t - \omega_n t) - \cos(\omega_n t + \omega_n t)] \tag{3B}$$

$$= \tfrac{1}{2} I_0^2[\cos(0) - \cos(2\omega_n t)] \tag{3C}$$

$$= \tfrac{1}{2} I_0^2[1 - \cos(2\omega_n t)] \tag{3D}$$

Now

$$\begin{aligned} \text{Instantaneous Square} \quad & \{i_0(t)\} = \tfrac{1}{2} I_0^2[1 - \cos(2\omega_n t)] \\ \text{Mean Square} \quad & \{i_0(t)\} = \tfrac{1}{2} I_0^2 \\ \text{Root Mean Square} \quad & \{i_0(t)\} = \sqrt{\tfrac{1}{2}} I_0 \end{aligned} \tag{4}$$

In a physical instrument the mean value will be determined through some kind of averaging process. If this process is not exact then the second harmonic term $I_0^2 \cos(2\omega_n t)$ will not vanish completely, and must be filtered out. A similar result occurs for calculations such as the instantaneous power:

$$\begin{aligned} p_0(t) &= v_0(t) i_0(t) \\ &= [V_0 \sin(\omega_n t)][I_0 \sin(\omega_n t - \varphi)] \\ &= \bar{p}_0 - \tfrac{1}{2} V_0 I_0 \cos(2\omega_n t - \varphi) \\ &= \bar{p}_0 - \tfrac{1}{2} V_0 I_0 \cos(2\omega_n t - \varphi) \end{aligned} \tag{5}$$

where φ is the angle by which $i_0(t)$ lags $v_0(t)$. The mean power $\bar{p}_0 = \tfrac{1}{2} V_0 I_0 \cos(\varphi)$ is the product of rms voltage $\sqrt{\tfrac{1}{2}} V_0$, rms current $\sqrt{\tfrac{1}{2}} I_0$, and the power factor $\cos(\varphi)$. The not very exact terminology “rms power” should be taken in this context.

Calculations for three phase circuits draw upon the further identity

$$\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y) \tag{6}$$

This, together with (1), shows that the second harmonic term cancels out for instantaneous total quantities in ideally balanced three phase circuits. This cancellation should not be relied upon under practical conditions, however.