Appendix C

Space Vector, Positive and Negative Sequence Vectors

This appendix relates to Chapter 3, §3.4.6.

A space vector (or a complex vector) for a three-phase quantity denoted by $\bar{x}(t)$ is chosen such that the projections of the vector in the three directions $0, -120^\circ$ and 120° give the three-phase instantaneous variables $x_a(t), x_b(t)$ and $x_c(t)$. The space vector can be calculated from the three-phase instantaneous quantities as follows:

$$\bar{x}(t) = T\begin{bmatrix} x_a(t) \\ x_b(t) \\ x_c(t) \end{bmatrix}$$

Where $T = \frac{2}{3}\begin{bmatrix} 1 & e^{-j\frac{2\pi}{3}} & e^{j\frac{2\pi}{3}} \end{bmatrix}$

The real and imaginary part of $\bar{x}(t)$ are usually called α and β components of the space vector:

$$\bar{x}(t) \equiv \bar{x}_{\alpha\beta}(t) = x_\alpha(t) + x_\beta(t)$$

It's possible to define the three-phase system by symmetrical components, i.e., positive, negative and zero sequence quantities:

$$\begin{bmatrix} x_a(t) \\ x_b(t) \\ x_c(t) \end{bmatrix} = X_P\begin{bmatrix} \cos(\omega t + \varphi_p) \\ \cos(\omega t + \varphi_p - \frac{2\pi}{3}) \\ \cos(\omega t + \varphi_p + \frac{2\pi}{3}) \end{bmatrix} + X_n\begin{bmatrix} \cos(\omega t + \varphi_n) \\ \cos(\omega t + \varphi_n + \frac{2\pi}{3}) \\ \cos(\omega t + \varphi_n - \frac{2\pi}{3}) \end{bmatrix} + X_0\begin{bmatrix} \cos(\omega t + \varphi_0) \\ \cos(\omega t + \varphi_0) \\ \cos(\omega t + \varphi_0) \end{bmatrix}$$

The space vector of above equation is obtained through applying operator T:

$$\begin{aligned} \bar{x}(t) &= T\begin{bmatrix} x_a(t) \\ x_b(t) \\ x_c(t) \end{bmatrix} \\ &= X_P e^{j(\omega t + \varphi_p)} + X_n e^{-j(\omega t + \varphi_n)} \\ &\overset{\Delta}{=} \bar{X}_p(t) + \bar{X}_n(t) \end{aligned}$$

The positive and negative vectors can be obtained as follows: If we assume that within a small time period τ the values of X_p, φ_p, X_n, φ_n are unchanged, we have:

$$\begin{cases} \bar{x}(t) = \bar{X}_p(t) + \bar{X}_n(t) \\ \bar{x}(t-\tau) = e^{-j\omega\tau}\bar{X}_p(t) + e^{+j\omega\tau}\bar{X}_n(t) \end{cases}$$

Solving the above equation system yields:

$$\bar{X}_p(t) = \frac{-j}{2\sin(\omega\tau)}\left[e^{+j\omega\tau}\,\bar{x}(t) - \bar{x}(t-\tau)\right]$$

$$\bar{X}_n(t) = \frac{+j}{2\sin(\omega\tau)}\left[e^{-j\omega\tau}\,\bar{x}(t) - \bar{x}(t-\tau)\right]$$

The above relationships can further be simplified if we set $\omega\tau = \frac{\pi}{2}$ or $\tau = \frac{T}{4}$.

$$\bar{X}_p(t) = \frac{1}{2}\left[\bar{x}(t) + j\,\bar{x}(t - \frac{T}{4})\right]$$

$$\bar{X}_n(t) = \frac{1}{2}\left[\bar{x}(t) - j\,\bar{x}(t - \frac{T}{4})\right]$$